PHYC/ECE 463 Advanced Optics I
Fall 2007
Homework #4, Due Wednesday Sept. 19

1. Metal Optics
Plot the surface reflectivity R ($=|r|^2$) versus wavelength (λ) for a metal having $\omega_p = 4\text{x}10^{15}$ rad/sec and τ=25 femtosecond (10^{-15} sec.). Assume normal incidence. Under white-light illumination, describe the color of the reflected (or scattered) light from this metal surface. *(4 pts.)*

2. Reflection: Problem 2.22 (K&F) *(3 pts.) (hint: angle of incidence!)*

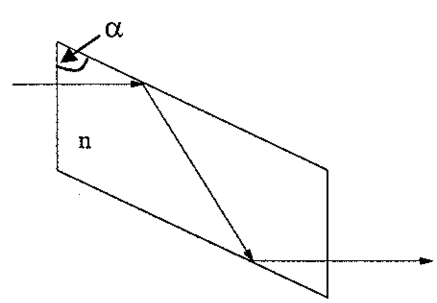
3. TIR *(10 pts.)*
1. (a) Show that the phase difference $\Delta=\phi_\pi-\phi_\sigma$ in total internal reflection from a glass-air interface can be given by:

$$\tan(\frac{\Delta}{2}) = \frac{\cos\theta\sqrt{\sin^2\theta - 1/n^2}}{\sin^2\theta}$$

(where $n=n_{glass}/n_{air}$)

(b) For a given glass with refractive index n, what is the largest phase difference (Δ), and at what incident angle θ?

(c) In a Fresnel rhomb, as shown below, Δ_{total} (upon two reflections) should be $\pi/2$. Determine the angle α when n=1.55.



(d) In constructing a Fresnel rhomb, what restriction is imposed on the material's refractive index..

4. FTIR: Problem 2.29 (K&F) *(3 pts.)*

1-Plot the surface reflectivity R ($=|r|^2$) versus wavelength (λ) for a metal having $\omega_p = 4\times10^{15}$ rad/sec and τ=25 femtosecond (10^{-15} sec.). Assume normal incidence. Under white-light illumination, describe the color of the reflected (or scattered) light from this metal surface. *(4 pts.)*

$\omega p := 4\cdot10^{15}$ $\quad$ $\tau := 25\cdot10^{-15}$ $\quad$ $c := 3\cdot10^{8}$ $\quad$ $i := \sqrt{-1}$

$\lambda p := 2\cdot\pi\cdot\dfrac{c}{\omega p}$ $\quad$ $\lambda p = 4.712\cdot10^{-7}$ $\quad$ $a := \omega p\cdot\tau$ $\quad$ $a = 100$

$\lambda := .2\cdot10^{-6}, .22\cdot10^{-6} .. 10\cdot10^{-6}$ wavelength range in meter

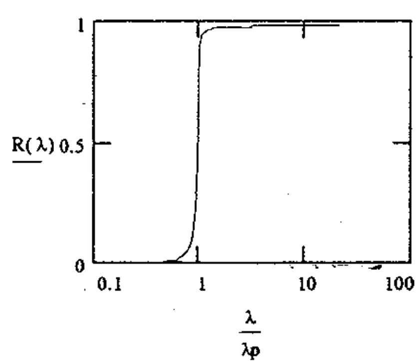
m

(1) $\quad \kappa r(\lambda) := 1 - \dfrac{1}{\left(\dfrac{\lambda p}{\lambda}\right)^2 + \dfrac{1}{a^2}}$ $\qquad$ $\kappa i(\lambda) := \left[\dfrac{1}{\left(\dfrac{\lambda p}{\lambda}\right)^2 + \dfrac{1}{a^2}}\right]\cdot\dfrac{\lambda}{\lambda p\cdot a}$

$\eta(\lambda) := \sqrt{\kappa r(\lambda) - i\cdot\kappa i(\lambda)}$ $\qquad$ $n(\lambda) := \mathrm{Re}(\eta(\lambda))$ $\qquad$ $k(\lambda) := \mathrm{Im}(\eta(\lambda))$

(1) $\quad R(\lambda) := \dfrac{(n(\lambda)-1)^2 + k(\lambda)^2}{(n(\lambda)+1)^2 + k(\lambda)^2}$

(1)



(1) Since λp=470 nm represents blue-green light, the reflected light will therefore be yellowish (green+red).

4-Problem 2.22 (K&F) *(2 pts.)*

Let's plot $R_{\sigma,\pi}$ versus θ_i assuming a complex refractive index considering two extreme cases as follows.
Case I: large real part, small imaginary part

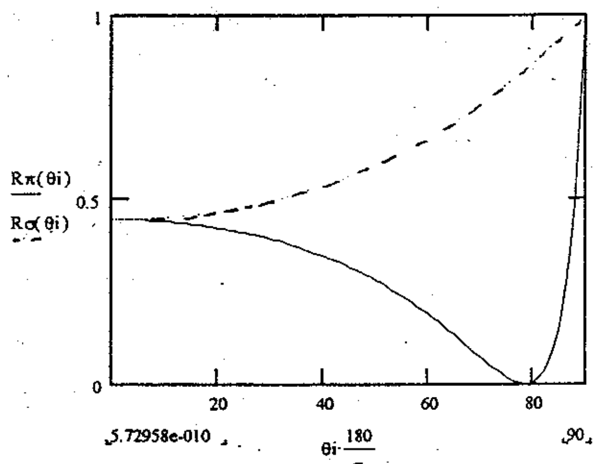
$n := 5 + i \cdot .1$ $\qquad\qquad n = 5 + 0.1j$

$$\theta i := 0.00000000001, \frac{\pi}{180} .. \frac{\pi}{2}$$

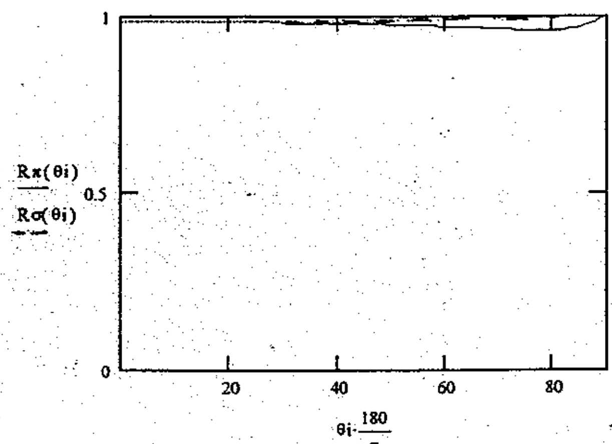
$$\theta t(\theta i) := \text{asin}\left(\frac{\sin(\theta i)}{n}\right)$$

$$r\pi(\theta i) := \frac{\tan(\theta i - \theta t(\theta i))}{\tan(\theta i + \theta t(\theta i))} \qquad r\sigma(\theta i) := \frac{-\sin(\theta i - \theta t(\theta i))}{\sin(\theta i + \theta t(\theta i))}$$

$$R\pi(\theta i) := \text{Re}(r\pi(\theta i))^2 + \text{Im}(r\pi(\theta i))^2 \qquad R\sigma(\theta i) := \text{Re}((r\sigma(\theta i)))^2 + \text{Im}(r\sigma(\theta i))^2$$



Case II, n=.1+i5 (samll real part, large imaginary part)



Thus, if R vs θi varies drastically (particulary for π polarization), the high R is due to n, otherwise, like metals, it is due to large imaginary part *k*.

3 (a) Show That $\Delta = \Phi_\pi - \Phi_\sigma$ in TIR is given by

$$\tan\left(\frac{\Delta}{2}\right) = \frac{\cos\theta\sqrt{\sin^2\theta - 1/n^2}}{\sin^2\theta} \quad \text{where } n \Rightarrow \frac{n_{glass}}{n_{air}}$$

From Eqn. 2-83 KF (Page 85)

$$\frac{\Phi_\sigma}{2} = \tan^{-1}\gamma \qquad \frac{\Phi_\pi}{2} = \tan^{-1}\frac{n^2}{n'^2}\gamma = \tan^{-1}n^2\gamma$$

$$\text{where } \gamma = \frac{\sqrt{\sin^2\theta - 1/n^2}}{\cos\theta}$$

$$\tan\left(\frac{\Delta}{2}\right) = \tan\left(\frac{\Phi_\pi - \Phi_\sigma}{2}\right) = \frac{\tan(\Phi_\pi/2) - \tan(\Phi_\sigma/2)}{1 + \tan(\Phi_\pi/2)\tan(\Phi_\sigma/2)}$$

$$= \frac{n^2\gamma - \gamma}{1 + n^2\gamma^2} = \frac{\gamma(n^2 - 1)}{1 + n^2\gamma^2}$$

$$\therefore \tan\left(\frac{\Delta}{2}\right) = \frac{\sqrt{\sin^2\theta - 1/n^2}\,(n^2-1)}{\cos\theta\left(1 + \frac{n^2\sin^2\theta - 1}{\cos^2\theta}\right)} = \frac{\cos\theta\sqrt{\sin^2\theta - 1/n^2}}{\sin^2\theta}$$

(b) what is Δ_{max} ?

We find $\left[\tan\left(\frac{\Delta}{2}\right)\right]_{max}$ by setting $\frac{d}{d\theta}(\quad) = 0$

$$\frac{d}{d\theta}\left(\tan\frac{\Delta}{2}\right) = \frac{-\sin^3\theta\sqrt{\sin^2\theta - \frac{1}{n^2}} + \sin^3\theta\cos^2\theta\left(\sin^2\theta - \frac{1}{n^2}\right)^{-\frac{1}{2}} - 2\sin\theta\cos^2\theta\sqrt{\sin^2\theta - \frac{1}{n^2}}}{\sin^4\theta}$$

$$= \frac{\sin^2\theta\left(\frac{1}{n^2} - \sin^2\theta\right) + \sin^2\theta\cos^2\theta - 2\cos^2\theta\left(\sin^2\theta - \frac{1}{n^2}\right)}{\sin^3\theta\left(\sin^2\theta - \frac{1}{n^2}\right)^{1/2}}$$

$$= \frac{\frac{\sin^2\theta}{n^2} - \cancel{\sin^4\theta} + \sin^2\theta - \cancel{\sin^4\theta} - 2\sin^2\theta + \frac{2}{n^2} + 2\cancel{\sin^4\theta} - \frac{2\sin^2\theta}{n^2}}{(\qquad)}$$

$$= \frac{\frac{2}{n^2} - \frac{\sin^2\theta}{n^2} - \sin^2\theta}{(\qquad)} = 0$$

$$\Rightarrow \sin^2\theta\left(\frac{1+n^2}{n^2}\right) = \frac{2}{n^2} \Rightarrow \sin^2\theta = \frac{2}{1+n^2}$$

$$\boxed{\sin\theta = \sqrt{\frac{2}{1+n^2}}}$$

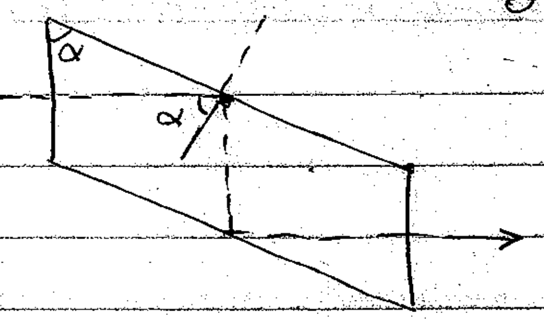
Substitute $\sin\theta$ into eqn. for $\tan\frac{\Delta}{2}$.

$$\left(\tan\frac{\Delta}{2}\right)_{max} = \sqrt{\frac{n^2-1}{n^2+1}} \times \frac{\sqrt{\frac{2}{1+n^2} - \frac{1}{n^2}}}{2/(1+n^2)} = \frac{n^2-1}{2n}$$

$$\boxed{\left(\tan\frac{\Delta}{2}\right)_{max} = \frac{n^2-1}{2n}}$$

(c) find α when $n = 1.55$ for the Fresnel rhomb.

$\theta = \alpha$



$$\Delta_{total} = \frac{\pi}{2} \Rightarrow \frac{\Delta}{2} \text{ per reflection} = \frac{\pi}{8}$$

$$\tan\frac{\pi}{8} = \frac{\cos\theta\sqrt{\sin^2\theta - \frac{1}{(1.55)^2}}}{\sin^2\theta} = 0.414$$

let $x = \sin^2\theta \Rightarrow 0.414 = a$, $b = \frac{1}{(1.55)^2}$

$$x^2(a^2+1) - x(b+1) + b = 0$$

$$x = \frac{b+1 \pm \sqrt{(b+1)^2 - 4b(a^2+1)}}{2(a^2+1)} = \frac{1.416 \pm \sqrt{2.006 - 1.950}}{2.343}$$

$$= \frac{1.416 \pm 0.236}{2.343} = \begin{cases} 0.7053 \\ 0.503 \end{cases}$$

$\sin^2\theta = 0.503 \Rightarrow \theta_1 = 45.2^\circ$

$\sin^2\theta = 0.705 \Rightarrow \theta_2 = 57.1^\circ$

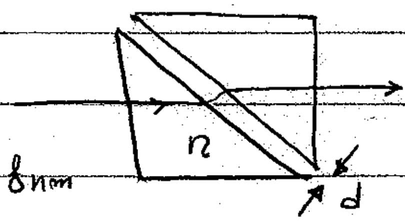
(d) what is the material restrictions for Fresnel rhomb?

From Section (b) $\left(\tan\frac{\Delta}{2}\right)_{max} = \frac{n^2-1}{2n} \geq \tan\frac{\pi}{8}$

$n^2 - 0.828n - 1 \geq 0 \quad n \geq \tan\frac{\pi}{8} + \sqrt{\left(\tan\frac{\pi}{8}\right)^2 + 1} = 1.4966$

$$\boxed{n \geq 1.4966}$$

4 - (KF- 2.29)



$n = 1.6$ $\qquad \lambda_0 = 632.8\,nm$

$\theta = 45°$

$$T_\pi = 2e^{-2d/\delta}(1 - \cos 2\varphi_\pi) = 0.5$$

what is $\underline{\underline{d}}$?

$$\delta = \frac{\lambda_0}{2\pi\sqrt{n^2\sin^2\theta - 1}} = \frac{632.8}{2\pi\sqrt{\frac{(1.6)^2}{2} - 1}} = 190.33\ nm$$

$$\tan\left(\frac{\varphi_\pi}{2}\right) = \frac{n^2\sqrt{n^2\sin^2\theta - 1}}{n\cos\theta} = 1.197$$

$$\varphi_\pi = 100.26° \Rightarrow \cos(2\varphi_\pi) = -0.9365$$

$$1 - \cos 2\varphi_\pi = 1.936$$

$$\Rightarrow e^{-2d/\delta} = \frac{0.5}{2 \times 1.936} \simeq 0.129$$

$$\Rightarrow d/\delta \approx 1.02 \qquad \boxed{d \approx \delta \approx 190\ nm}$$